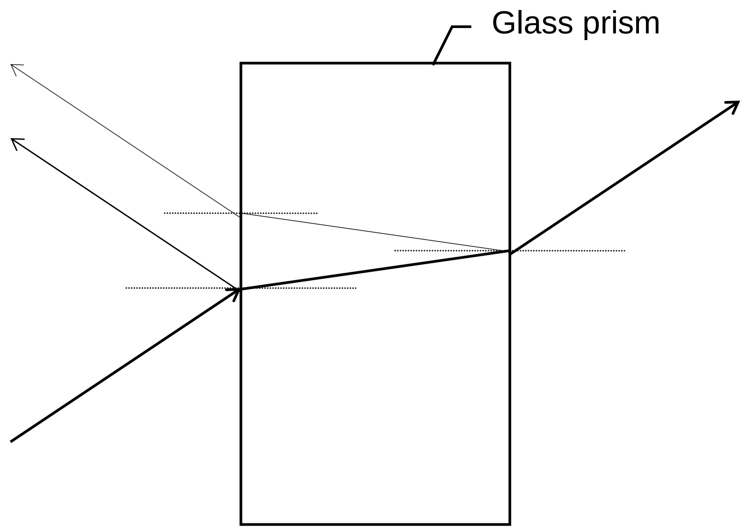
Physics 30 Lesson 8 Refraction of Light

Refer to Pearson pages 666 to 674.

I. Reflection and refraction of light

At any interface between two different mediums, some light will be reflected and some will be refracted, except in certain cases which we will soon discover. When problem solving for refraction, we usually ignore the reflected ray.



II. Index of refraction

The fastest that light can travel is in a vacuum ($c = 3.00 \times 10^8$ m/s). In other substances, the speed of light is always slower. The *index of refraction* is a ratio of the speed of light in vacuum with the speed of light in the medium:

$$\text{index of refraction (n)} = \frac{\text{speed in vacuum (c)}}{\text{speed in medium (v)}}$$

$$\mathbf{n = \frac{c}{v}}$$

Some common indices of refraction are:

Substance	Index of refraction (n)
vacuum	1.0000
air	1.0003
water	1.33
ethyl alcohol	1.36
quartz (fused)	1.46
glycerine	1.47
Lucite or Plexiglas	1.51
glass (crown)	1.52
sodium chloride	1.53
glass (crystal)	1.54
ruby	1.54
glass (flint)	1.65
zircon	1.92
diamond	2.42

Notice that the index of refraction (n) is always greater than or equal to 1 and that it has no units.

Example 1

The index of refraction for crown glass was measured to be 1.52. What is the speed of light in crown glass?

$$n = \frac{c}{v}$$

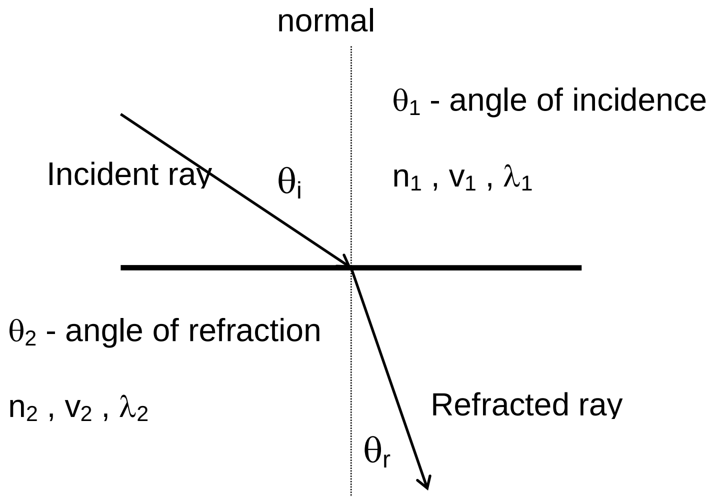
$$v = \frac{c}{n}$$

$$v = \frac{3.00 \times 10^8 \ ^{m}/_{s}}{1.52}$$

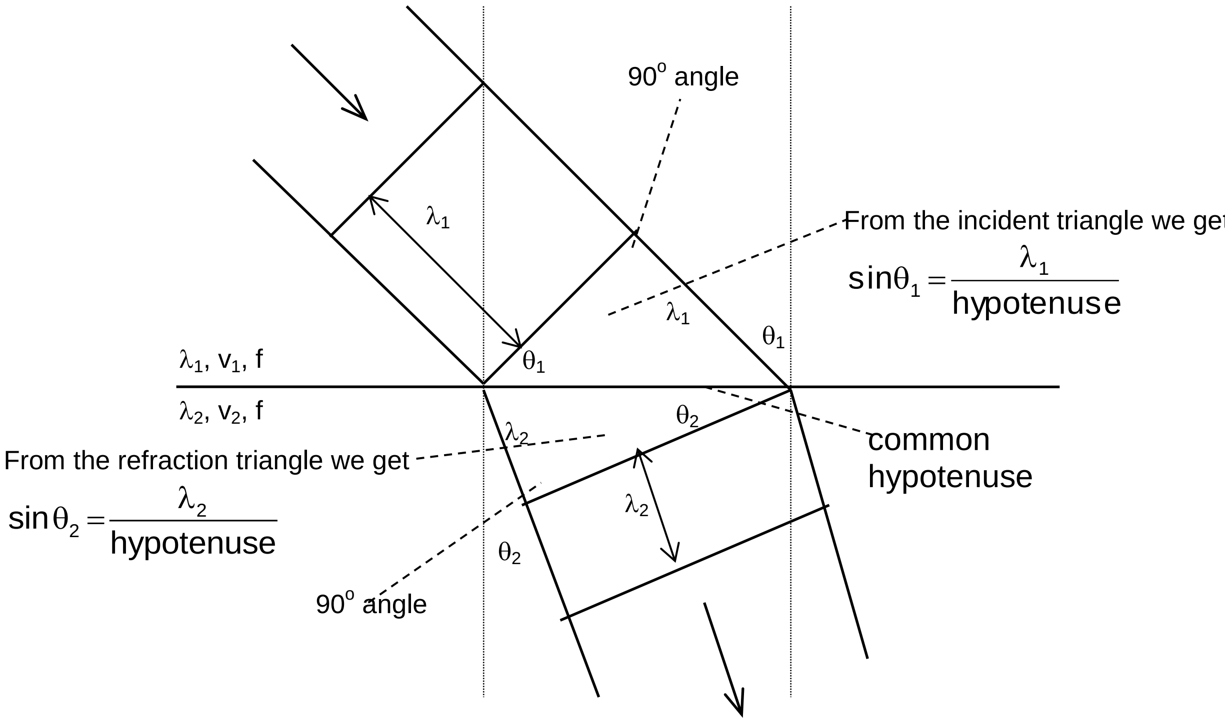
$$v = \mathbf{1.97 \times 10^8 \ ^{m}/_{s}}$$

III. Law of Refraction (Snell's Law)

Refraction is the change in *speed*, *wavelength* and *direction* of light caused by a change in medium. For example, when light passes from air into water, the speed decreases, the wavelength decreases, and the light ray bends in toward the normal.



In 1621, Willebrord Snell, a Dutch mathematician, determined the relationship between the angles, wavelengths and speeds of refracted waves. These relationships are referred to as **Snell's law** or the **Law of Refraction**. We will now derive Snell's law. The diagram represents a wave travelling from a fast medium into a slow medium.



Dividing the equations we get $\dfrac{\sin\theta_1}{\sin\theta_2} = \dfrac{\dfrac{\lambda_1}{\text{hypotenuse}}}{\dfrac{\lambda_2}{\text{hypotenuse}}} = \dfrac{\lambda_1}{\lambda_2}$

We have $\dfrac{\sin\theta_1}{\sin\theta_2} = \dfrac{\lambda_1}{\lambda_2}$

Using the universal wave equation $v = f\lambda \rightarrow \lambda = \dfrac{v}{f}$ we can derive another relationship:

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{\frac{v_1}{f}}{\frac{v_2}{f}} = \frac{v_1}{v_2}$$

For light we also have another relationship that we can include. Since $v = \dfrac{c}{n}$

$$v_1 = \frac{c}{n_1} \quad \text{and} \quad v_2 = \frac{c}{n_2} \quad \text{then} \quad \frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}$$

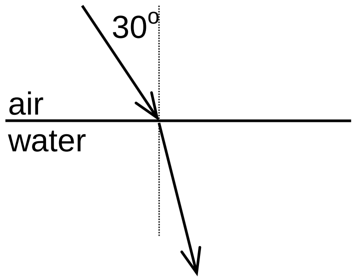
Snell's Law (i.e. **The Law of Refraction**) is:

$$\boxed{\frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}}$$

and we can use any pairing that we desire.

Example 2

If light has an angle of incidence of 30° when travelling from air into water, what is the angle of refraction?



$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

$$\sin\theta_2 = \frac{n_1\sin\theta_1}{n_2}$$

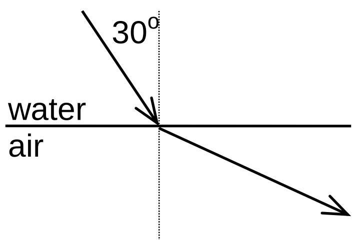
$$\theta_2 = \sin^{-1}\left(\frac{n_1\sin\theta_1}{n_2}\right)$$

$$\theta_2 = \sin^{-1}\left(\frac{1.00\sin 30}{1.33}\right)$$

$$\theta_2 = \mathbf{22^o}$$

Example 3

If light has an angle of incidence of 30° when travelling from water into air, what is the angle of refraction?



$$\sin\theta_2 = \frac{n_1\sin\theta_1}{n_2}$$

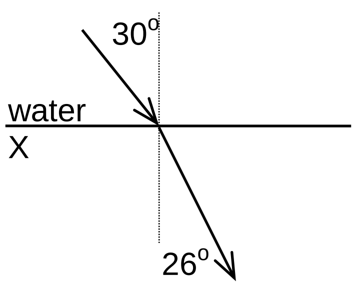
$$\theta_2 = \sin^{-1}\left(\frac{n_1\sin\theta_1}{n_2}\right)$$

$$\theta_2 = \sin^{-1}\left(\frac{1.33\sin 30}{1.00}\right)$$

$$\theta_2 = \mathbf{41.9^o}$$

Example 4

If light has an angle of incidence of 30° and an angle of refraction of 26° when travelling from water into substance X, what is the index of refraction for X?



$$n_2 = \frac{n_1\sin\theta_1}{\sin\theta_2}$$

$$n_2 = \frac{1.33\sin 30}{\sin 26}$$

$$n_2 = \mathbf{1.52}$$

Example 5

If the speed of light in air is 3.00 x 10^8 m/s, what is the speed of light in glass (n_{glass} = 1.50)?

$$\frac{v_1}{v_2} = \frac{n_2}{n_1}$$

$$v_2 = \frac{n_1 v_1}{n_2}$$

$$v_2 = \frac{1.00(3.00\times10^8\ m/s)}{1.50}$$

$$v_2 = \mathbf{2.00\times10^8\ m/s}$$

Example 6

If light has a wavelength of 750 nm in air, what is the wavelength of light in diamond ($n_{diamond}$ = 2.42)?

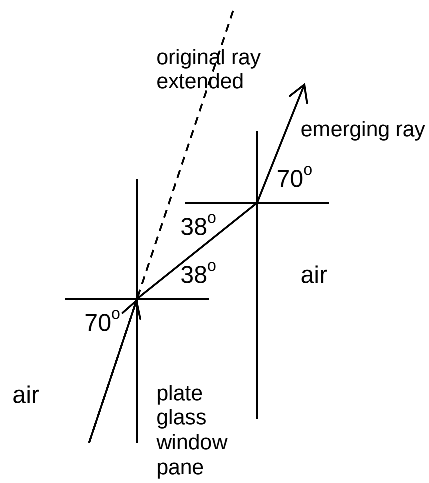
$$\frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1} \qquad \lambda_2 = \frac{n_1 \lambda_1}{n_2}$$

$$\lambda_2 = \frac{1.00(750\,\text{nm})}{1.50}$$

$$\lambda_2 = \mathbf{310\,nm}$$

IV. Special problems

A. Parallel sides



When light passes through a medium with parallel sides, the original angle of incidence always equals the final angle of refraction. The internal angles are also the same. Note that the original ray is parallel with the emerging ray.

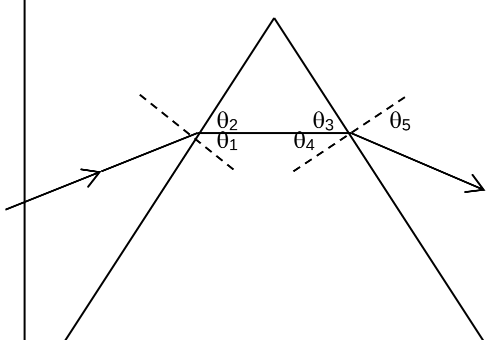
B. Triangular prisms

A classic problem is when light refracts through a triangular prism. Since the sides are not parallel, the internal angles of refraction and incidence will not be the same.

Example 7

If light enters an equilateral glass prism (n = 1.50) with an angle of incidence of 45°, what is the angle of refraction as the beam emerges from the prism?

Note that an equilateral triangle has three 60° angles.



$\sin\theta_1 = \frac{1.00}{1.50}\sin 45^\circ \qquad \theta_1 = 28.12^\circ$

$\theta_2 = 90^\circ - \theta_1 = 90^\circ - 28.12^\circ = 61.88^\circ$

$\theta_3 = 180^\circ - (60^\circ + \theta_2) = 180^\circ - 121.88^\circ = 58.12^\circ$

$\theta_4 = 90^\circ - \theta_3 = 90^\circ - 58.12^\circ = 31.88^\circ$

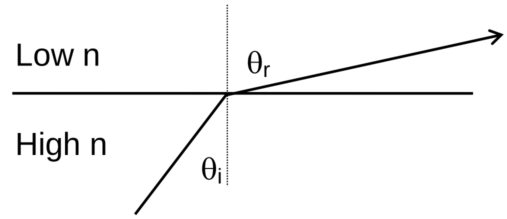
$\sin\theta_5 = \frac{1.50}{1.00}\sin 31.88^\circ$

$\theta_5 = \mathbf{52.4^\circ}$

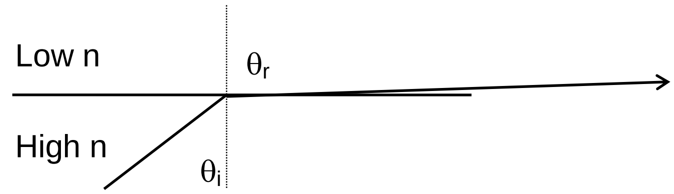
V. Total internal reflection

Refer to Pearson pages 672 to 674.

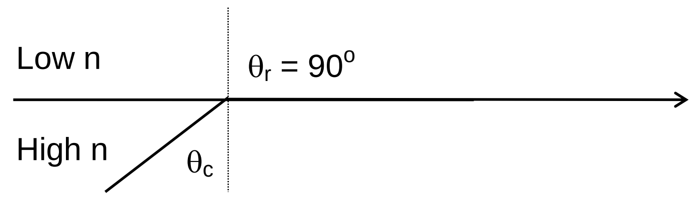
When a light ray passes from a more optically dense medium (high n, low v) to a less optically dense medium (low n, high v), the angle of refraction (θ_r) is greater than the angle of incidence (θ_i).



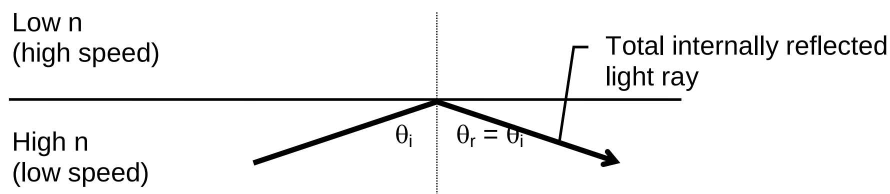
As the angle of incidence is increased, the angle of refraction becomes larger until finally θ_r approaches 90^o.

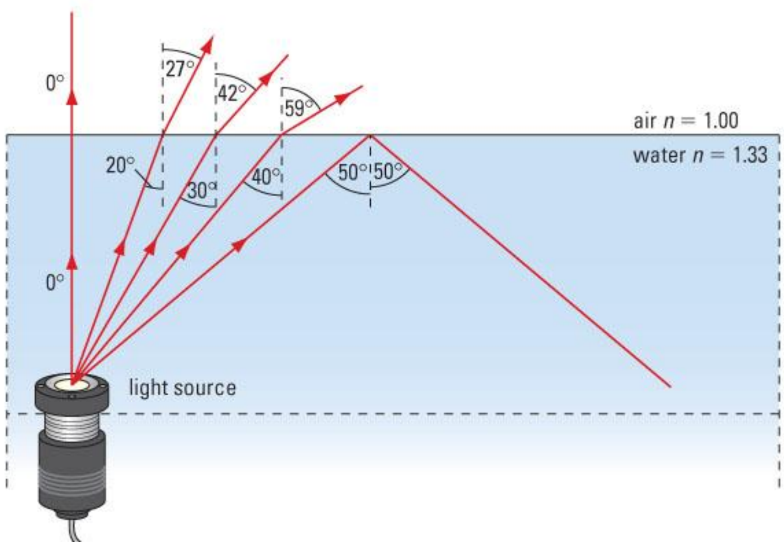


At the angle of incidence called the **critical angle** (θ_c) the angle of refraction = 90^o.



At angles beyond the critical angle, refraction can no longer occur – the result is **total internal reflection** which obeys the law of reflection. In other words, at angles beyond the critical angle the boundary between the media acts as a mirror surface.





The properties of internal reflection are used in fibre optic technologies and in optical instruments like cameras, microscopes, binoculars, etc.

Note that **total internal reflection occurs only when light travels from low to high speed media**. It does not occur when light travels from high to low speed media.

Example 8

What is the critical angle for a water and air interface?

For the critical angle $\theta_2 = 90^o$:

$$\sin\theta_c = \frac{n_2 \sin\theta_2}{n_1}$$

$$\theta_c = \sin^{-1}\left(\frac{n_2 \sin\theta_2}{n_1}\right)$$

$$\theta_c = \sin^{-1}\left(\frac{1.00 \sin 90}{1.33}\right)$$

$$\theta_c = \mathbf{48.8^o}$$

Example 9

If light makes an angle of incidence of 60^o when travelling from diamond into water, what is the angle of refraction produced?

$$\theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right)$$

$$\theta_2 = \sin^{-1}\left(\frac{2.42 \sin 60}{1.33}\right)$$

$$\theta_2 = \textbf{error}$$

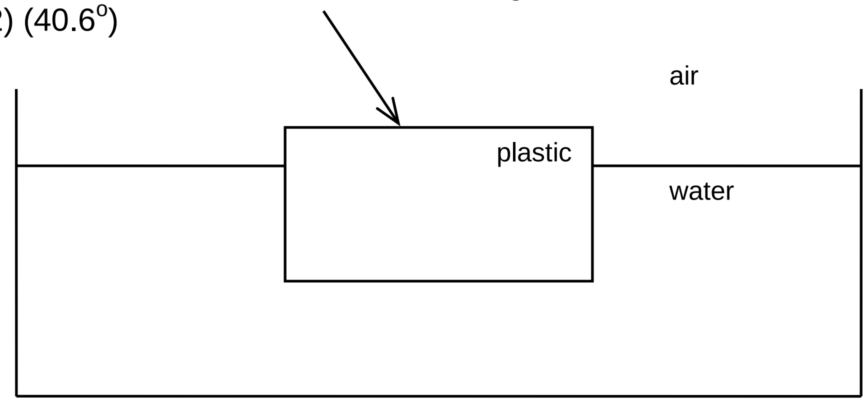
Note that this error result does not indicate that you have made a mistake in calculation. The result means that no refraction is possible resulting in **total internal reflection**.

VI. Practice problems

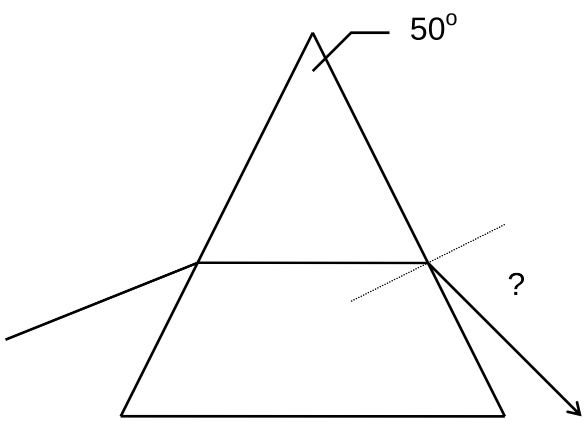
1. A ray of light has an angle of incidence of 24^o from air into water. What is the angle of refraction? (Refer to the table of refractive indices at the beginning of this lesson.) (17.8^o)

2. A light ray is travelling from crown glass into air. The angle of incidence is 20^o. What is the angle of refraction? (31.3^o)

3. The angle of refraction in air is 60^o . Find the angle of refraction in the water. ($n_{plastic}$= 1.52) (40.6^o)



4. For the following prism (n = 1.50) find the indicated angle if the angle of incidence is 40^o. (38.7^o)

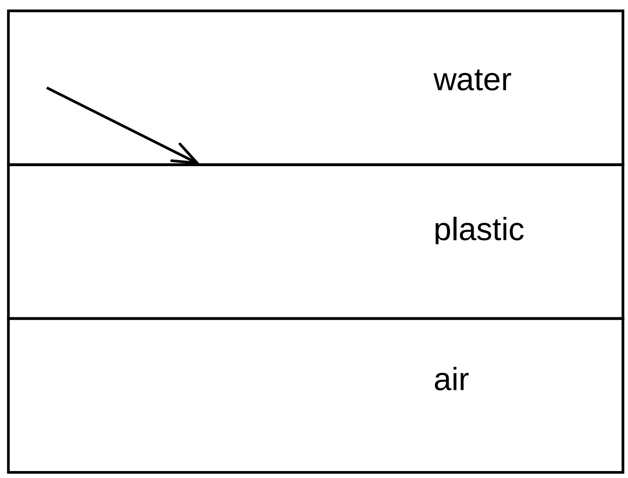


5. To successfully spear a fish, you must aim below the apparent position of the fish. Explain.

6. What is the critical angle when light emerges from glass (n = 1.50) into air? (41.8^o)

7. The critical angle between glass and water is 56.2^o. What is the index of refraction for the glass? (1.60)

8. The angle of refraction in water is 60^o . If $n_{plastic}$= 1.62 trace the ray path through the system below.



VII. Hand-in assignment

1. The speed of light in a certain plastic is 2.0×10^8 m/s. What is the refractive index of the plastic? (1.5)

2. The index of refraction of crown glass for violet light is 1.53. and for red light 1.52. Assuming that the velocity of light in a vacuum is 3.00×10^8 m/s, what are the speeds of violet light and red light in crown glass? (1.96×10^8 m/s, 1.97×10^8 m/s)

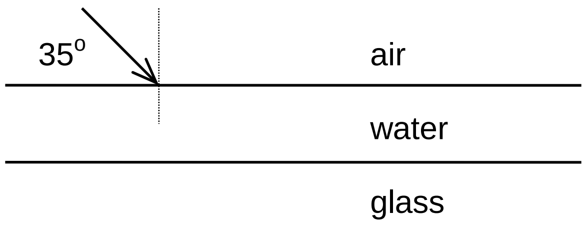
3. A beam of light strikes the surface of water with an incident angle of 60^o. Some of the light reflects off the water and some refracts into the water. If water has an index of refraction of 1.33, determine the angles of reflection and refraction. (60^o, 41^o)

4. A wave travelling from air to glass n = 1.52) has an angle of incidence of 30°. What is the angle of refraction? (19°)

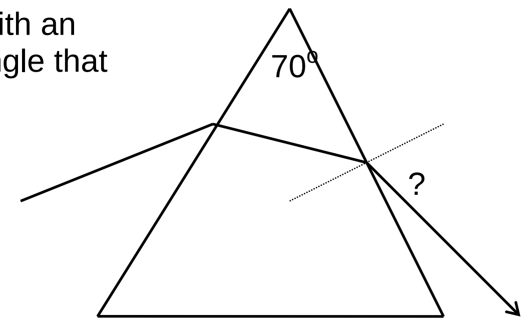
5. If the angle of incidence is 20° and the angle of refraction is 10°, what is the index of refraction of the material if the wave started in air? (1.97)

6. What is the wavelength of light in water if the wavelength in air is 570 nm? (429 nm)

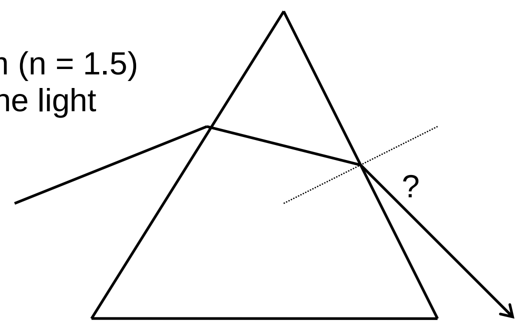
7. A ray of light enters from air to water and then into glass as shown in the diagram. Find the angle of refraction in glass. (n_{water} = 1.33, n_{glass} = 1.50) (33°)



8. Light strikes the 70° glass prism (n = 1.51) with an angle of incidence of 50.0°. Calculate the angle that the light leaves the prism. (73.9°)

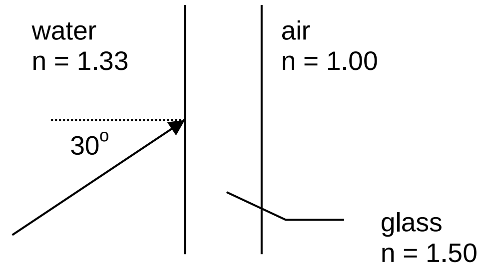


9. Light is incident on an equilateral Lucite prism (n = 1.5) at an angle of 35°. Calculate the angle that the light leaves the prism. (66°)



10. Freddie the fish shines his flashlight upward from under water.
    - A. If Freddie shines his flashlight at an incident angle of 20°, at what angle does the light leave the water? (27°)
    - B. Beyond what critical angle will the light no longer be able to leave the water? (48.8°)

11. From inside an aquarium a ray of light is directed at the glass so the angle of incidence, in water, is 30°. (a) Determine the angle of refraction when the ray emerges from the glass into the air. (b) If the angle of incidence in the water is 52° at what angle will the rays emerge from the glass? (42°, no ray emerges)



12. An underwater swimmer looks up toward the surface of the water on a line of sight that makes an angle of 25° with a normal to the surface of the water. What is the angle of incidence in air for the light rays that enter the swimmer's eyes? (34°)

13. You tape a penny to the inside bottom of a cup and move your eye away from the cup to just that point where you can no longer see the penny over the rim of the cup. If someone slowly adds water to the cup, the penny comes into view. Using diagrams and a word description, explain why. (Pennies and styrofoam cups are available. Seeing is believing! Try it!)

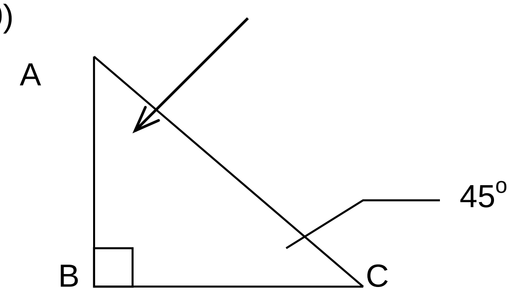
14. Hot air rises over a heated stove element. Explain why the wall behind the stove appears to shimmer.

15. The critical angle from rock salt into air is 40.5°. What is the index of refraction for rock salt. (1.54)

16. The critical angle for a certain liquid-air surface is 61°. What is the liquid's index of refraction? (1.14)

17. The refractive indices of diamond and crown glass are 5/2 and 3/2 respectively. What is the critical angle between diamond and glass? (37°)

18. A ray of light enters AC along the normal. Trace the subsequent ray path. (n = 1.50)



19. A bright point source of light is placed at the bottom of a 20 cm deep pool of water, n = 1.33. What is the area of the circle of light which is seen at the surface of the water? What happened to the other light rays? (1634 cm^2)

20. A student performs an experiment where she measures the angles of incidence and refraction for a particular type of glass. She obtains the following results:

angle of incidence	angle of refraction
0°	0°
10°	6°
20°	12°
30°	18°
40°	23°
50°	28°
60°	32°

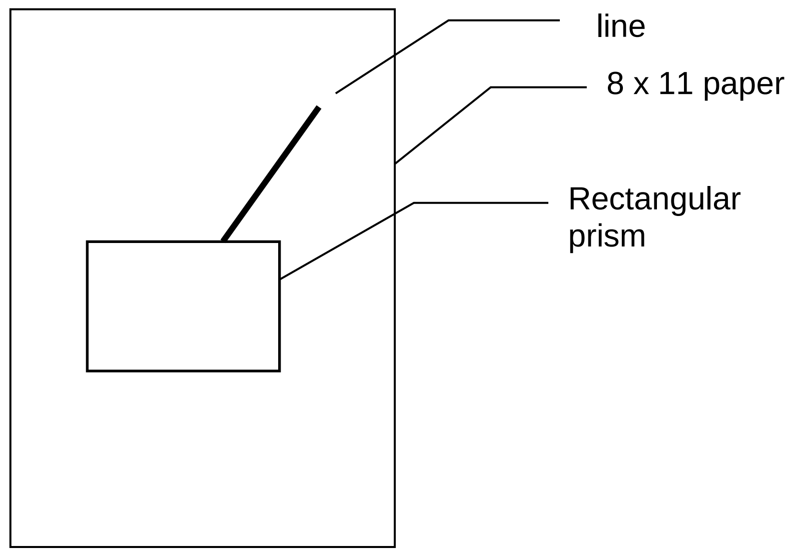
Using an appropriate graphing technique, calculate the index of refraction for the glass.

Activity 1 – Index of Refraction

Procedure:

1. Working with <u>one</u> partner, place a piece of paper on the table and place the rectangular prism as shown in the diagram below. With a pencil, trace around the rectangular prism. Then draw a line that meets the rectangular prism at an angle of incidence of 50^o.

Top View



2. Place the laser pointer so that the light runs down the pencil line. Look for the emergent ray as it leaves the prism and draw a pencil line for the emergent ray.

3. Draw in the ray path through the prism.

4. At the point where light enters the prism draw in a normal to the prism surface. Using a protractor measure the angle of incidence and the angle of refraction at the interface.

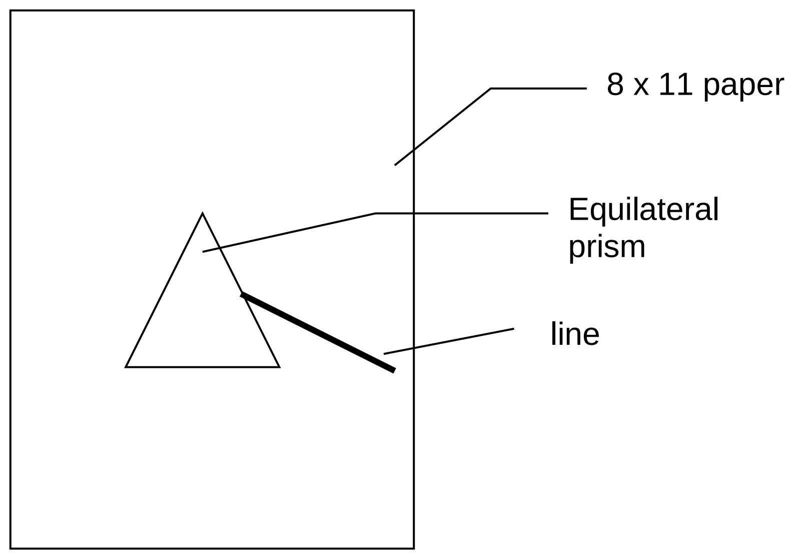
5. Calculate the index of refraction for the rectangular prism.

Activity 2 – Equilateral Triangle Prism

Procedure:

1. Working with one partner, place a piece of paper on the table and arrange an equilateral triangle prism as shown in the diagram below. With a pencil, trace around the prism and draw a line as shown.

Top View



2. Place the laser pointer so that the light runs down the pencil line. Look for the emergent ray as it leaves the prism and draw a pencil line for the emergent ray.

3. Draw all of the resulting ray paths including the rays through the prism and those reflected from and within the prism.

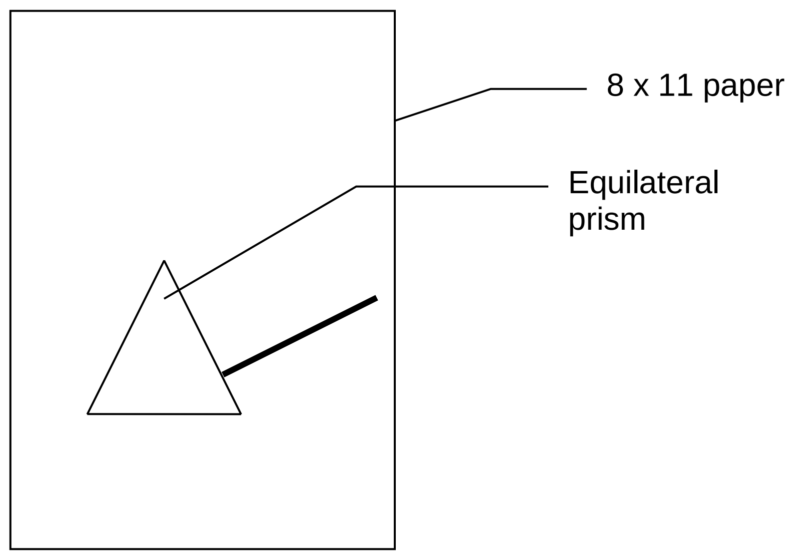
4. For each ray, explain how and why the ray is emerging where it is.

Activity 3 – Equilateral Triangle Prism (Total Internal Reflection)

Procedure:

1. Working with <u>one</u> partner, place a piece of paper on the table and arrange a equilateral triangle prism as shown in the diagram below. With a pencil, trace around the prism. Use a protractor to draw a line at 90^{o} to the prism face. (Note: Make sure that you do not draw the line from the centre of the face of the triangular prism. The line should be offset from centre.)

Top View



2. Point the laser pointer directly down the line. Draw a pencil line in for the emergent ray.

3. Draw in the ray path through the prism. Explain the resulting ray path.